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Radiative Decay of Vector Meson V $\rightarrow$ Pγ in the Spinor Strong Interaction Theory

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The spinor strong interaction theory recently developed is applied to the radiative decay of a two-quark vector meson into pseudoscalar meson V $\rightarrow$Pγ. Expression of the decay rate Γ is derived in this first-principle theory without assumption and free parameter. The ratio $\Gamma(D^{*0} \rightarrow D^0\gamma)/\Gamma(D^{*+} \rightarrow D^+\gamma)$ is correctly predicted. The orders of magnitude of the radiative decay rates of B*, D*, K*, and ρ estimated from this expression are consistent with data. Very fast mesons have a smaller size then do mesons at rest, similar to Lorentz contraction in laboratory space.

1. INTRODUCTION

In the current literature, low-energy mesonic theories are based upon phenomenological Lagrangians [1, 2]. QCD-oriented, nonrenormalizable, chiral perturbation theories are applied to light mesons [2, 3]. When the meson contains a very heavy quark, a new spin-flavor symmetry emerges from QCD [4, 5] and can be incorporated to apply to heavy mesons [6, 7]. In spite of the vast literature on these subjects, specific and systematic predictions on the radiative decay of vector mesons V $\rightarrow$ Pγ are scanty [8].

This may be due to the phenomenological nature of these theories in which the meson fields are local. Since the meson has finite size, much physics is lost by neglecting its extension and compensating for it by introducing parameters. A comparison of these Lagrangians to the nonlocal Lagrangian of the spinor strong interaction theory [9; hereafter denoted by I] has been given in the introduction and Section 8 of the accompanying paper [10; hereafter denoted by III] and will not be repeated here.

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Reference I is concerned with stationary phenomena of mesons at rest. It has been extended to treat weak decay of pseudoscalar mesons [11; hereafter denoted by II].

The purpose of this paper is to modify and extend II to apply to the radiative decay of two-quark vector mesons V $\to$ Pγ. In the process, paper I is also extended to apply approximately to slowly moving heavy pseudoscalar mesons. In Section 2, the required action integral is given and its first-order part identified with the aid of expressions from earlier work reproduced in Appendix A. The decay amplitude is then derived in Section 3. The decay rate is derived in Section 4. It is further reduced to an order-of-magnitude estimate of this rate by introducing the perturbed wave functions, arising from the slow motion of heavy pseudoscalar-mesons, obtained from a dimensional analysis in Appendix B. In this appendix, it is indicated on dimensional grounds that the size of the meson decreases with increasing momentum, similar to Lorentz contraction in laboratory space. The decay rate is applied to the ratios of D* $\to$ Dγ decays in Section 5. Order-of-magnitude estimates of the decay rates for B*, D*, K*, and ρ are found to be consistent with data. Comparison with earlier work is made.

2. ACTION FOR V $\to$ Pγ DECAY

Appendix A gives the starting equations mostly collected from earlier papers on this theory. Substituting (A6a), (A6b), (A7), and (A9) into (A4) and making use of (A6c) and (A5b) leads to

$$S_M = \int d^4X\, d^4x\, \frac{1}{4}\Big\{[(\partial_{\mathrm{I}}^{\dot{b}a} + i(1-a)q_p A^{\dot{b}a}(X))\chi_a^{\dot{e}}][(\partial_{\mathrm{II}\dot{e}f} - iaq_r A_{\dot{e}f}(X))\chi_{\dot{b}}^{f}]$$
$$+ [(\partial_{\mathrm{II}}^{\dot{b}a} - iaq_r A^{\dot{b}a}(X))\psi_a^{\dot{e}}][(\partial_{\mathrm{I}\dot{e}f} + i(1-a)q_p A_{\dot{e}f}(X))\psi_{\dot{b}}^{f}]$$
$$+ 2(\Phi_p(x) - M_m^2)\psi_d^{\dot{c}}\chi_{\dot{c}}^{d} + \text{h.c.}\Big\} \tag{2.1}$$

which is similar to the action (II 2.4) for K $\to \mu\nu$ and provides the present starting point.

The equivalent of (II 4.1) is here

$$(\partial_{\mathrm{I}}^{\dot{b}a}\chi_a^{\dot{e}})(\partial_{\mathrm{II}\dot{e}f}\chi_{\dot{b}}^{f}) = \partial_{\mathrm{I}}^{\dot{b}a}\chi_a^{\dot{e}}\partial_{\mathrm{II}\dot{e}f}\chi_{\dot{b}}^{f} - \chi_a^{\dot{e}}\partial_{\mathrm{I}}^{\dot{a}b}\partial_{\mathrm{II}\dot{e}f}\chi_{\dot{b}}^{f} \tag{2.2}$$

We substitute (2.2) and other terms of the same type into (2.1). Following II Section 4, terms of the type of the last term in (2.2) together with the $\Phi_P - M_m^2$ term in (2.1) vanish since they form the terms in the zeroth-order meson equations (A1). The remaining terms in (2.1) are of type (i) consisting of terms of the kind of the first term on the right of (2.2), type (ii) consisting of terms linear in the q's, and terms quadratic in q. These type (ii) terms represent the electromagnetic interaction and therefore are regarded as first-order perturbations. Other ordering of the terms in (2.1) is entirely analogous to that given by (II 2.6). Following the discussion below (II 4.4), the type (ii) terms here likewise balance off the type (i) terms, which therefore are also of first order. This is in agreement with the conventional S-matrix theory mentioned at the end of II Section 4.

3. DECAY AMPLITUDE

Equations (I 6.7) and (I 6.8) show that the meson equations (A1) reduce in the rest frame to two classes of solutions representing the pseudoscalar and vector mesons, which will be denoted by the subscripts $J = 0$ and 1, respectively.

With the help of (I 6. 1a), (I 6.3), and (I 6.5), the meson wave functions are decomposed as follows:

$$\psi^{a\dot{b}}(x_{\rm I}, x_{\rm II}) = \sum_J \sum_{\vec{K}} b_{J\vec{K}} \exp\left[-iE_{J\vec{K}}X^0 + i\vec{K}_J\vec{X} + i\omega_{J\vec{K}}x^0\right](\delta^{a\dot{b}}\psi_{J\vec{K}}(\vec{x}) - \vec{\sigma}^{a\dot{b}}\vec{\psi}_{J\vec{K}}(\vec{x})) \tag{3.1}$$

and a similar equation with $\psi \rightarrow \chi$. Here, $E_{J\mathbf{K}}$ is the energy of the meson with momentum $\mathbf{K}_J$ and $\omega_{J\mathbf{K}}$ is the relative energy of the quarks. The expansion (3.1) holds only for free mesons for which the confining potential $\Phi'_{PJr}(x)$, prior to its specialization to $\Phi'_{PJr}(r)$ of (III 2.1c), vanishes so that the meson wave equation (B1) is linear (see III Section 2.1). For the decaying meson, let

$$b_{J\vec{K}} \rightarrow a_{J\vec{K}} + a^{(1)}_{J\vec{K}}(X^0) \tag{3.2}$$

by analogy to II (3.1). Here $a_{J\mathbf{K}}$ is the annihilation operator for a meson with J and $\mathbf{K}$, and $a^{(1)}_{J\mathbf{K}}(X^0)$ is the corresponding first-order decay amplitude and varies slowly with the time X^0.

The photon of the decay can be represented as

$$\vec{A}(X) = \sum_{\vec{K}_r} (2E_r\Omega)^{-1/2} \sum_T \vec{e}_T a_T(\vec{K}_r) \exp(-iE_r X^0 + i\vec{K}_r\vec{X}) + \text{c.c.} \tag{3.3}$$

where Ω is a large normalization volume, $(E_r, \mathbf{K}_r)$ is the four-momentum of the photon, $\mathbf{e}_T$ is the unit polarization vector in the transverse directions $T = 1$ and 2 perpendicular to $\mathbf{K}_r$, and $a_T(\mathbf{K}_r)$ is the corresponding annihilation operator. The time component A^0 has been put to zero.

As in (II 4.3b) and (II 4.3c), let

$$\left|i\right\rangle = \left|V(\bar{\mathbf{K}}_1 = 0)\right\rangle, \qquad \left\langle f\right| = \left\langle P(\bar{\mathbf{K}}_0),\ \gamma_T(\bar{\mathbf{K}}_r)\right|$$

$$a_{10}\left|i\right\rangle = \left|0\right\rangle, \qquad a^*_{o\bar{K}_o} a^*_T(\bar{\mathbf{K}}_r)0\rangle = \left|f\right\rangle \tag{3.4}$$

where $\left|i\right\rangle$ and $\left\langle f\right|$ denote the initial and final states, respectively. The last relation in (3.4) holds because the sum of four-momenta of P and γ is the same as that of $V(\mathbf{K}_1 = 0)$ in X space, irrespective of their behavior in the relative x space.

Collecting all type (i) terms in (2.1), placing them between $\left\langle f\right|$ and $\left|i\right\rangle$, and making use of (3.1), (3.1) with $\psi \to \chi$, (3.2), (3.4), and (A3b) leads to

$$\left\langle f\right| \int d^4X\, d^4x \frac{1}{4} \{\partial_{\mathrm{I}}^{ba}\chi_a^{\dot e}\, \partial_{\mathrm{II}\dot e f}\chi_b^{f} + \partial_{\mathrm{II}}^{ba}\psi_a^{\dot e}\, \partial_{\mathrm{I}\dot e f}\psi_b^{f} + \partial_{\mathrm{II}fe}\chi_b^{f}\, \partial_{\mathrm{I}}^{b\dot a}\chi_a^{e} + \partial_{\mathrm{I}ef}\psi_b^{f}\, \partial_{\mathrm{II}}^{b\dot a}\psi_a^{e}\}\left|i\right\rangle$$

$$= -i\frac{1}{2} E_{10} S_{fi} \int d^3\bar{\mathbf{X}} \int dx^0\, d^3\bar{\mathbf{x}}\, \left|\bar{\psi}_{10}(\bar{\mathbf{x}})\right|^2 \tag{3.5a}$$

$$S_{fi} = \left\langle f\right| a_{10}^{(1)*}(X^0 \to \infty) a_{10}\left|i\right\rangle \tag{3.5b}$$

Here, integration over X^0 has been carried out with the boundary condition $a_{10}^{(1)}\ (X^0 \to -\infty) = 0$. Also,

$$\psi_{10} = \chi_{10} = 0, \qquad \bar{\psi}_{10} = \bar{\chi}_{10} = \hat{r}\psi_1, \qquad \hat{r} = \bar{\mathbf{x}}/r, \qquad r = \left|\bar{\mathbf{x}}\right| \tag{3.6}$$

for V at rest according to (I 6.8) and (I 8.2) ff.

The sum of the type (ii) terms in (2.1) is

$$i\frac{1}{8} \int d^4X\, d^4x \Big\{ q_p[(\partial_{\mathrm{II}}^{ba}\psi_a^{\dot e}) A_{\dot e f}(X)\psi_b^{f} + A^{ba}(X)\chi_a^{\dot e}\partial_{\mathrm{II}\dot e f}\chi_b^{f}]$$

$$- q_r[(\partial_{\mathrm{I}}^{ba}\chi_a^{\dot e}) A_{\dot e f}(X)\chi_b^{f} + A^{ba}(X)\psi_a^{\dot e}\, \partial_{\mathrm{I}\dot e f}\psi_b^{f}] + \mathrm{h.c.} \Big\} \tag{3.7}$$

where $a = 1/2$ has been chosen in (2.1), in agreement with the consideration preceding (4.1) below. Sandwiching (3.7) between $\left\langle f\right|$ and $\left|i\right\rangle$ and making use of (A3b), (3.1)–(3.4), and (3.6) leads to

$$\frac{1}{4}(2E_r\Omega)^{-1/2} \int d^3\bar{\mathbf{X}} \exp(-i(\bar{\mathbf{K}} + \bar{\mathbf{K}}_r)\bar{\mathbf{X}}) 2\pi\delta(E_{0\bar{K}} + E_r - E_{10})$$

$$\times \int dx^0\, d^3\bar{\mathbf{x}} \frac{1}{2}$$

$$\times \psi_1 \left\{ \begin{array}{l} q_p[(\bar{\mathbf{e}}_T\hat{r})(E_{0\bar{K}}\psi^*_{0\bar{K}} - E_{10}\chi^*_{0\bar{K}} - \bar{\mathbf{K}}\bar{\psi}^*_{0\bar{K}}) + (\bar{\mathbf{e}}_T \times \hat{r})(\bar{\mathbf{K}} \times \bar{\psi}^*_{0\bar{K}}) + iI_p] \\ -q_r[(\bar{\mathbf{e}}_T\hat{r})(E_{0\bar{K}}\chi^*_{0\bar{K}} - E_{10}\psi^*_{0\bar{K}} - \bar{\mathbf{K}}\bar{\chi}^*_{0\bar{K}}) + (\bar{\mathbf{e}}_T \times \hat{r})(\bar{\mathbf{K}} \times \bar{\chi}^*_{0\bar{K}}) + iI_r] \end{array} \right\}$$

$$+\mathrm{h.c.} \tag{3.8}$$

$$I_p = (\bar{e}_T \times \hat{r})(\bar{K}\chi^*_{0\bar{K}} - E_{0\bar{K}}\bar{\chi}^*_{0\bar{K}} - 2\bar{\partial} \times \bar{\chi}^*_{0\bar{K}}) + (\bar{e}_T\hat{r})(\bar{\partial}\bar{\chi}^*_{0\bar{K}})$$
$$+ E_{10}\hat{r}(\bar{e}_T \times \bar{\psi}^*_{0\bar{K}}) + 2(\bar{e}_T\bar{\psi}^*_{0\bar{K}})(\partial\psi_1/\partial r)/\psi_1$$
$$I_T = (\bar{e}_T \times \hat{r})(\bar{K}\psi^*_{0\bar{K}} - E_{0\bar{K}}\bar{\psi}^*_{0\bar{K}} + 2\bar{\partial} \times \bar{\psi}^*_{0\bar{K}}) - (\bar{e}_T\hat{r})(\bar{\partial}\bar{\psi}^*_{0\bar{K}})$$
$$+ E_{10}\hat{r}(\bar{e}_T \times \bar{\chi}^*_{0\bar{K}}) - 2(\bar{e}_T\bar{\chi}^*_{0\bar{K}})(\partial\psi_1/\partial r)/\psi_1 \tag{3.9}$$

where $\mathbf{K} \equiv \mathbf{K}_0$ for brevity. The decay amplitude S_{fi} is obtained by equating (3.8) to the negative of (3.5a).

4. DECAY RATE FORMULA AND ESTIMATES

In the rest frame and in the absence of orbital excitation, the pseudoscalar meson wave function is a singlet and depends only upon the quark–antiquark distance r. For mesons in motion, however, the wave functions satisfy the full (A1) and (A2), which couple the singlet and triplet parts, and spherical symmetry in $\mathbf{x}$ for the rest frame case is lost. In motion, a pseudoscalar meson is thus no longer represented only by a singlet, the time component of a four-vector, but by a four-vector in $\mathbf{X}$ and $\mathbf{x}$ (see Section 3.3 of III). Note that x is not an observable since quarks are not seen, but observable results characterizing the mesons depend upon it. Therefore, x may take on the role of a "hidden variable" mentioned in the literature.

Mesons in motion are considered in Appendix B and described by (B1), which has not been solved generally. Therefore, (3.8) and (3.9) cannot be evaluated for arbitrary K. For nonrelativistic pseudoscalar mesons, however, their wave functions can be expanded in powers of K or ε_0 of (B5). The perturbed wave functions are determined by (B8) to order ε_0 and by (B9) ff. to order ε_0^2.

4.1. Decay Rate Formula for Slow Mesons and Estimate

The treatment that follows will therefore be limited to slowly moving pseudoscalar mesons according to (B5). Equation (B6) is substituted into (3.8) and (3.9) and only terms to order ε_0^2 are kept. Here,

$$\psi^*_{0\bar{K}}(\bar{x}) \to \psi_{00}(r) + \psi_{02}(\bar{x}), \qquad \bar{\psi}^*_{0\bar{K}} \to \bar{\psi}_{01}(\bar{x}), \qquad \psi \to \chi \tag{4.1}$$

Equation (B8) shows that $\bar{\psi}_{01}$ and $\bar{\chi}_{01}$ are real. Therefore, (3.9) is real and the iI_p and iI_r terms in (3.8) drop out. Since $E_{0\mathbf{K}} \neq E_{10}$, (B1d) requires that $\omega_{0\mathbf{K}} \neq \omega_{10}$ in (3.8) observing (3.1). This will lead to an inconsistency in the relative time x^0 dependence of (3.8) relative to that of (3.5). Therefore, $\omega_{J\mathbf{K}} = 0$ is set, which is consistent with (B9) ff. and (B3). Equation (B1d) now becomes $a = 1/2$, which has been used in (3.7). With (B7), (3.8) to order ε_0^2 is equated to the negative of (3.5a) and the result is put in the form

$$S_{fi} = i\pi(2E_r\Omega)^{-1/2}\ E_{10}^{-1}\delta(E_{0\bar{K}} + E_r - E_{10})\left(\int d^3\bar{x}\ \psi_1^2(r)\right)^{-1}$$

$$\times\left\{\begin{array}{l}\int d^3\bar{x}\ \psi_1(r)(\bar{e}_T\hat{r})[q_p(E_{0\bar{K}}\psi_{00}(r) - E_{10}\chi_{00}(r)) + q_r(E_{10}\psi_{00}(r) - E_{0\bar{K}}\chi_{00}(r))] + \\ \int d^3\bar{x}\ \psi_1(r)\frac{1}{2}\left[\begin{array}{l}(q_p + q_r)[(\bar{e}_T \times \hat{r})(\bar{K} \times (\bar{\psi}_{01} - \bar{\chi}_{01})) \\ + (\bar{e}_T\hat{r})(-\bar{K}(\bar{\psi}_{01} - \bar{\chi}_{01})) + (E_{0\bar{K}} + E_{10})(\psi_{02} - \chi_{02})] \\ + (q_p - q_r)[\bar{e}_T \times \hat{r})(\bar{K} \times (\bar{\psi}_{01} + \bar{\chi}_{01})) \\ + (\bar{e}_T\hat{r})(-\bar{K}(\bar{\psi}_{01} + \bar{\chi}_{01})) + (E_{0\bar{K}} - E_{10})(\psi_{02} + \chi_{02})]\end{array}\right]\end{array}\right\} \tag{4.2}$$

where $\mathbf{K}_r = -\mathbf{K}$ has been set.

The first integral is of order ε_0 and vanishes when integration over the angles is carried out. The decay rate is given by (II 5.4) with (II 6.4). With (4.2), it reads

$$\Gamma_2(\mathrm{V} \rightarrow \mathrm{P}\gamma) = \sum_{\text{final states}} |S_{fi}|^2/T_d$$

$$= 2\frac{1}{32\pi^2 E_{10}^2}\int d^3\bar{K}\ \delta(E_{0\bar{k}} + E_r - E_{10})\frac{K^4}{E_r E_{00}^2}$$

$$\times[(q_p + q_r)^2 Q_{\mathrm{I}+}^2 + (q_p - q_r)^2 Q_{\mathrm{I}-}^2] \tag{4.3}$$

$$Q_{\mathrm{I}\pm} = \left[\int d^3\bar{x}\ \psi_1^2(r)\right]^{-1}\int d^3\bar{x}\ \psi(r)\frac{E_{00}}{2K^2}$$

$$\times[(\bar{e}_T \times \hat{r})(\bar{K} \times (\bar{\psi}_{01} \mp \bar{\chi}_{01})) + (\bar{e}_T\hat{r})(-\bar{K}(\bar{\psi}_{01} \mp \bar{\chi}_{01})$$

$$+ (E_{10} \pm E_{0\bar{k}})(\psi_{02} \mp \chi_{02}))] \tag{4.4}$$

which holds to order ε_0^2. Here, T_d denotes a long time period during which all such decays occur. The cross product $(q_p + q_r)*(q_p - q_r)$ term has been dropped because the physical results must remain unchanged if $p \leftrightarrow r$, i.e., if quark I (II) has flavor r (p) instead of p (r).

The decay rate (4.3), however, cannot be evaluated because the perturbed wave functions in (4.4) have not been computed according to Appendix B.5. In the following, an estimate of (4.3) based upon the dimensional analysis approximation in Appendix B.5 will be given in the limit (B11) of heavy pseudoscalar mesons. Inserting (B14)–(B17) into (4.4) leads to

$$Q_{\mathrm{I}+} = \left[\int d^3\bar{x}\ \psi_1^2(r)\right]^{-1}[-1 - 4E_{00}(E_{0\bar{K}} + E_{10})/d_m^2]I_b \tag{4.5a}$$

$$I_b = \int d^3\bar{x}\ \psi_1(r)\psi_{00}(r)(\bar{e}_T'\hat{r}) \tag{4.5b}$$

$$Q_{\mathrm{I}-} = 0 \tag{4.5c}$$

where

$$\bar{\bar{e}}'_{\mathrm{T}} = (\hat{X}, \hat{Y}, 0) \perp \bar{\bar{K}}$$

and X and Y are laboratory coordinates. Equation (4.5b) again vanishes after integration over the angles. The decay rate (4.3) therefore also vanishes to order ε_0^2. This is in part due to the approximate nature of the dimensional analysis, which removes the angular dependence in (B14) and (B17) (see end of Appendix B.5). Such dependence can appear in higher orders of ε_0. Due to (B8a), the ε_0^3 terms do not contribute to (4.4), but ε_0^4 in (B6) will. In some of these terms, x, or y is expected to be present at least linearly due to the cross terms in (B1). With such a term in (4.4), integral over the angles will no longer vanish. Let $T = 1$ represent $\mathbf{e}_T$ in the X direction; then

$$\int d^3\bar{\bar{x}}\,(\bar{\bar{e}}_T\hat{r})\hat{x} = \frac{1}{3}\int d^3\bar{\bar{x}} \tag{4.6}$$

This term corresponds to the dipole transition picture of Section 4.3.

The dimensional analysis approximation of Appendix B.5 has not been carried out to order ε_0^4. To obtain an estimate to this order, the ε_0^2 order result (4.5), modified by including (4.6), will simply be multiplied by a factor ε_2 of order ε_0^2 so that

$$I_b \to I_{b2} = \frac{\varepsilon_2}{3}\int d^3\bar{\bar{x}}\;\psi_1(r)\psi_{00}(r) \tag{4.7}$$

Since both $\bar{\bar{\psi}}_{01}$ of order ε_0 and ψ_{02} of order ε_0^2 enter (4.4), an estimate combining both types of terms is

$$\varepsilon_2 = \left|\frac{\psi_{01}}{\psi_{00}}\right|\sqrt{\left|\frac{\psi_{02}}{\psi_{00}}\right|} \tag{4.8}$$

where (B5), (B14), and (B17) have been employed.

The integrals in (4.5a) and (4.7) can be evaluated using the free meson wave functions (III 2.2) and (III 2.3). One finds

$$\int d^3\bar{\bar{x}}\;\psi_1(r)\psi_{00}(r)\Big/\int d^3\bar{\bar{x}}\;\psi_1^2(r) = \sqrt{3/2}\;32/81 \tag{4.9a}$$

The photon energy $E_r = K_0$ and the pseudoscalar meson energy $E_{0\mathbf{K}}$ are determined by the δ-function in (4.3),

$$K_0 = E_{10}[1 - (E_{00}/E_{10})^2]/2, \qquad E_{0\bar{\bar{K}}} = E_{10} - K_0 \tag{4.9b}$$

Inserting (4.5), (4.7), and (4.9) into (4.3) and making use of (B10) leads to an order-of-magnitude estimate of the decay rate for slow pseudoscalar mesons to order ε_0^4,

$$\Gamma_4(\mathrm{V} \to \mathrm{P}\gamma) = \varepsilon_2 \Gamma_2(\mathrm{V} \to \mathrm{P}\gamma) \tag{4.10a}$$

$$\Gamma_2(\mathrm{V} \to \mathrm{P}\gamma) = \frac{128}{6561\pi}(q_p + q_r)^2 \frac{K_0^5}{E_{00}^2 E_{0\bar{K}}^2}\left(1 - \frac{E_{00}}{E_{10}}\right)^3 \times [(1 + 4E_{00}(E_{10} + E_{0\bar{K}}))/d_m^2]^2 \tag{4.10b}$$

$$\text{for} \quad \varepsilon_0 \ll 1 \quad \text{and} \quad d_m \ll E_{00}$$

where $d_m = 0.864$ Gev according to (III 2.5a).

4.2. Decay Rate Estimate for Fast Mesons

In Table I, the π and K mesons are relativistic in the ρ and K* decays. Therefore, the $\varepsilon_0 \ll 1$ results in Section 4.1 and Appendix B.5 no longer hold.

By analogy to the reverse of (4.1), setting

$$\begin{aligned} \psi_{02} &\to \psi_{0\bar{K}}(\bar{\bar{x}}) - \psi_{00}(r) \to \psi_{0\bar{K}}(\bar{\bar{x}}), \\ \bar{\bar{\psi}}_{01} &\to \bar{\bar{\psi}}_{0\bar{K}}(\bar{\bar{x}}), \qquad \psi \to \chi \end{aligned} \tag{4.11}$$

in (4.2)–(4.4), these will hold for all ε_0 values. Here, the $\psi_{00}(r)$ term vanishes upon integration over the angles in (4.4). The momentum K_0 for π and K is still low compared to d_m, so that the extremely relativistic case of Appendix B.2 does not apply. The scale of the meson wave functions is not $1/K_0$, as in (B4b), but is still of the order of $2/d_m$, as in $\psi_{00}(r)$ or (III 2.2a). The following order-of-magnitude estimate of the decay rate will be made.

For large K_0, the $\bar{\bar{\partial}}(\mathbf{K} \times \chi)$ type of terms in (B1) cannot be dropped so that the simple relations (B12a) and (B13a) no longer hold. Therfore, unlike (4.5c), Q_{1-} does not vanish.

The same reasoning leading to (4.6) is similarly assumed, but is now associated with large ε_0 values. In this case, the $\mathbf{e}_T \times \hat{\mathbf{r}}$ term in (4.4), using (4.11), vanishes for the x and y components of $\hat{r}$ because $\mathbf{e}_T \perp \mathbf{K}$. For the z component of $\hat{r}$, integration over the angles still yields zero or is strongly suppressed. Therefore, this $\mathbf{e}_T \times \hat{r}$ term is dropped. This term also did not contribute to (4.5) and is consistent with the physical picture of Section 4.3 below.

The $\mathbf{e}_T\, \hat{r}$ term in (4.4) together with (4.11) is now approximated by (4.5), including the modification implied in (4.6). Note that this extension of the $\varepsilon_0 \ll 1$ result to $\varepsilon_0 \gg 1$ cases is adopted for lack of a better estimate and is obviously very coarse. The order-of-magnitude estimate of the decay rate is

$$\Gamma_0(\mathrm{V} \rightarrow \mathrm{P}\gamma) = \left[1 + \left(\frac{q_p - q_r\, Q'_{1-}}{q_p + q_r\, Q'_{1+}}\right)^2\right]\Gamma_2(\mathrm{V} \rightarrow \mathrm{P}\gamma) \quad (4.12a)$$

$$Q'_{1+} = Q_{1+} I_{b2}/\varepsilon_2 I_b \quad (4.12b)$$

$$0 < |Q'_{1-}| \leq |Q_{1+}| \qquad \text{for} \quad \varepsilon_0 \gg 1 \text{ and } \pi \text{ in Table I} \quad (4.12c)$$

$$0 \leq |Q'_{1+}| < |Q_{1+}| \qquad \text{for} \quad \varepsilon_0 \leq 1 \text{ and K in Table I} \quad (4.12d)$$

The last relation is indicated by (4.5c), and (4.12c) is implied by the general form of (4.4) and (4.11), in which the "small components" $\psi_{0\mathbf{K}}$ and $\chi_{0\mathbf{K}}$ become nearly as large as the "large components" $\psi_{0\mathbf{K}}$ and $\chi_{0\mathbf{K}}$ for pseudoscalar mesons with large momenta.

4.3. Physical Picture

Consider as an example the D^{*0} decay in Fig. 1. The radii of the maximum and the average meson wave function amplitudes are obtained from the nonrelativistic free meson wave functions (III 2.2). The transition has a dominant electric dipole nature, in adition to the spin-flip or magnetic dipole transition considered conventionally [12]. A simple spin flip will not transform D* to D, which have different sizes.

In this picture, the direction $\mathbf{e}_\tau$ of the vector potential is parallel to the current $\mathbf{j}$ or to $\hat{r}$. This is reflected in that only the $\mathbf{e}_T\hat{r}$ term, but not the $\mathbf{e}_T \times \hat{r}$ term, enters (4.5), and hence the decay rates (4.10) and (4.12).

5. APPLICATION AND COMPARISON TO EARLIER WORK

5.1. Application

Table I shows the V $\rightarrow$ Pγ cases considered.

Consider first the D* decay, $(q_p + q_r)^2 = 4\pi/137$ times 16/9 and 1/9 for D^{*0} and D^{*+}, respectively. Further, Q_{1+} of (4.4) is almost the same for both decays, as is indicated by (4.5a). With the masses from ref. 13, (4.3) and (4.5c) lead to

$$\Gamma_2(D^{*0} \rightarrow D^0\gamma)/\Gamma_2\,(D^{*+} \rightarrow D^+\gamma) = 16.96 \approx \frac{16}{9}\frac{9}{1} \quad (5.1)$$

These may be compared to the data [13]

$$\frac{\Gamma(D^{*0} \rightarrow D^0\gamma)}{\Gamma(D^{*0} \rightarrow D^0\pi^0)}\frac{\Gamma(D^{*+} \rightarrow D^+\pi^0)}{\Gamma(D^{*+} \rightarrow D^+\gamma)} = \frac{38.1\%}{61.9\%}\frac{30.6\%}{1.1\,\pm\,^{2.1}_{0.7}\%} = 17.12 \quad (5.2)$$

In Table II of III, $\Gamma(D^{*0} \rightarrow D^0\pi^0)/\Gamma(D^{*+} \rightarrow D^+\pi^0)$ has been estimated to be

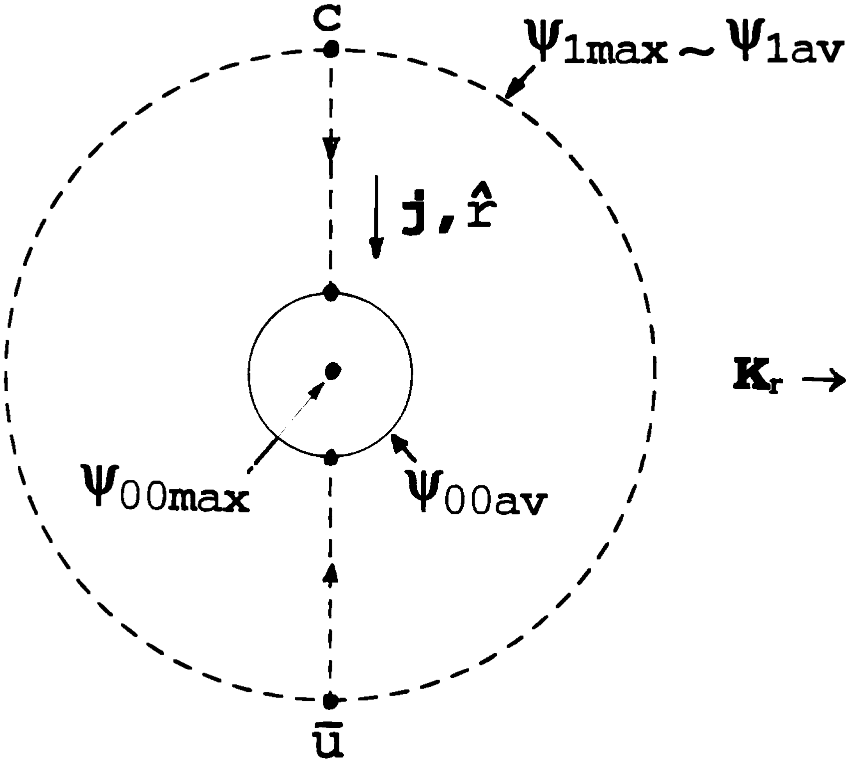

Fig. 1. Illustration of $D^{*0} \to D^0\gamma$. Here $\psi_{1\max}$ and $\psi_{1\mathrm{av}}$ are the maximum and average amplitudes, respectively, of the vector meson wave function ψ_{10}; $\psi_{00\max}$ and $\psi_{00\mathrm{av}}$ are the maximum and average amplitudes, respectively, of the pseudoscalar meson wave functions. The sizes are given by (III 2.6) and (III 2.5a). The much greater radius of the vector meson leads to a much greater potential energy $-d_m/r$ of (III 2.1b) than does the final-state pseudoscalar meson. The vector meson is therefore unstable and tends to reduce its potential energy, which is achieved by diminishing its size. Therefore, the quarks c and $\bar{u}$ move along the dashed line inward to a distance of the order of the diameter of a pseudoscalar meson. Similarly, the vector meson wave function peaking on the dashed circle collapses into the origin, where the pseudoscalar meson wave function has its maximum. One of the quarks flips its spin to convert D^{*0} to D^0 and gives rise to the spin of the photon. The quark movements along the spin direction of the D^{*0} give rise to a current $\mathbf{j}$ which gives off a photon with momentum $\mathbf{K}_r$. The resulting D^0 acquires a momentum $-\mathbf{K}_r$.

1.16. Inserting this value into (5.2) leads to $\Gamma(D^{*0} \to D^0\gamma)/\Gamma(D^{*+} \to (D^+\gamma)$ $= 19.9$, which is about 17% greater than the prediction of (5.1). However, there is a very large error margin of the branching ratio $1.1 \pm {}^{2.1}_{0.7}\%$ in (5.2). Agreement with (5.1) is restored if $1.1\% \to 1.29\%$ in (5.2), which is well within the error limits.

Note that (5.1) relies on the approximations (B5) and (B11), which are

Table I. Some Data and Order-of-Magnitude Estimates of the Decay Rate Γ for Vector Mesons V Decaying Radiatively into Pseudoscalar Mesons P[a]

V	B^{*0}	B^{*+}	D^{*0}	D^{*+}	K^{*0}	K^{*+}	ρ^0	ρ^+
P	B^0	B^+	D^0	D^+	K^0	K^+	π^0	π^+
$\epsilon_0 = K_0/E_{00}$	$0.0124 \ll 1$		$0.073 \ll 1$		$0.62 \leq 1$		$2.7 \gg 1$	
Γ (keV)	1.9×10^{-8}	0.5×10^{-8}	2.8×10^{-3}	0.17×10^{-3}	53.5	$\geq$14.5 <145	(732[b])	($\gg$ 70) ($\leq$700)
Equation		(4.10)			(4.12a,b,d)		(4.12a,b,c)	
Data (keV)	Dominant		<800	<1.44	116	50.3	119	67.8

[a] ε_0 is the ratio of the momentum to mass for P. The very large Γ values for ρ can be due to the extrapolation of the $\varepsilon_0 \ll 1$, $d_m \ll E_{00}$ estimate to $\epsilon_0 \gg 1$, $d_m \gg E_{00}$ for ρ and are indicated by the parentheses.

[b] Average for $\bar{u}u$ and $\bar{d}d$ contributions (see note *a* in Table 4 of ref. 12).

well and relatively well, respectively, satisfied for D. It does not make use of the dimensional analysis approximation (4.5a) itself and is therefore a relatively accurate prediction.

This is in contrast to the order-of-magnitude estimates of the decay rates in Table I, which are obtained from (4.10) based on such an approximation and from (4.12) based upon a still coarser approximation. The estimated D* and B* decay rates are much smaller than the observed upper limits. An experimental determination of these rates may therefore provide a clear test of the spinor strong interaction theory.

For K*, the experimental ratio $\Gamma(K^{*0} \rightarrow K^0\gamma)/\Gamma(K^{*+} \rightarrow K^+\gamma) = 2.31$ lies between the estimated ones, i.e., $3.7 \geq 2.31 > 0.37$. The upper limit 3.7 is obtained if $Q_{I-} = 0$, so that the $q_p - q_r$ term in (4.3) does not contribute. Thus, the $q_p - q_r$ term does seem to contribute and (4.5c) no longer holds. This may be consistent with the fact that $\varepsilon_0 \ll 1$ underlying (4.5) is violated by $\varepsilon_0 = 0.62$ in Table I. This is in contrast to the D* decay, for which (5.1) is nearly the ratio of the $(q_p + q_r)^2$ factor or 16; the $q_p - q_r$ term drops out in (4.3) due to (4.5c).

The estimated ρ decay rates are too large and may be due to the extrapolation and modification of the $\varepsilon_0 \ll 1$ and $d_m \ll E_{00}$ result of (4.5) to $\varepsilon_0 \gg 1$ and $d_m \gg E_{00}$ for ρ decay. The experimental ratio $\Gamma(\rho^0 \rightarrow \pi^0\gamma)/\Gamma(\rho^+ \rightarrow \pi^+\gamma) = 1.76$ again lies between the estimated ones, i.e., $10.5 > 1.76 \geq 1.05$. The upper limit 10.5 is obtained with $Q_{I-} = 0$ of (4.5c), so that the $q_p - q_r$ term in (4.3) does not contribute. However, the requirements underlying (4.5c) are violated in a still higher degree so that $|Q_{I-}/Q_{I+}|$ is still larger and approaches unity. This is represented by the lower limit 1.05, which is closer to the data.

In the sequence of D*, K*, and ρ decays, the pseudoscalar mesons get more relativistic, the Q_{I-}^2 term goes from 0 to nearly Q_{I+}^2, and the $(q_p + q_r)^2$

dependence of the D* decay rate is transformed to a $(q_p + q_r)^2 + (q_p - q_r)^2 = 2(q_p^2 + q_r^2)$ dependence for ρ.

5.2. Comparison to Earlier Work

The comparison in Section 8.3 of III made for the sister process V → PP holds here as well. In Table 2 of ref. 8 radiative decay rates of light mesons were obtained in the chiral limit and agree with data rather well. Nevertheless, the chiral Lagrangian used contains a large number of terms and a number of parameters have to be fixed by other data. Corrections due to departure from chirality have not been considered. To account for decays of heavy mesons, heavy quark symmetry [4, 5] has to be called upon and this gives rise to additional complications [6]. Further, Lagrangians of this type consist of local meson fields and cannot account for confinement, the $U(1)$ problem, the absence of Higgs bosons, etc.

These limitations are removed in the nonlocal Lagrangian in (2.1) of the spinor strong interaction theory [14, 11]. The decay rate (4.3) with (4.4), (4.11) has been derived from the Lorentz- and gauge-invariant action (2.1) without any assumption and without any free parameter to be fixed by other data. However, (B1) and (B2) determining the zeroth-order wave functions for mesons in motion are too complicated to solve. Approximations and assumptions have been introduced to estimate these wave functions needed in the decay rate expression. In spite of these gross approximations, the estimated Γ in Table I do not contradict and are in order-of-magnitude agreement with data.

APPENDIX A. EQUATIONS FROM EARLIER WORK

The basic meson equations together with related transformations are given by (5.4), (5.5), (4.12), (6.2), and (A2) of I:

$$\partial_{\mathrm{I}}^{a\dot{b}} \partial_{\mathrm{II}\dot{e}f}\, \chi^f_{\dot{b}}(x_{\mathrm{I}}, x_{\mathrm{II}}) \xi^p_r(z_{\mathrm{I}}, z_{\mathrm{II}}) = (\Phi_p(x_{\mathrm{I}}, x_{\mathrm{II}}) - M^2_m) \psi^a_{\dot{e}}(x_{\mathrm{I}}, x_{\mathrm{II}}) \xi^p_r(z_{\mathrm{I}}, z_{\mathrm{II}})$$

$$\partial_{\mathrm{I}\dot{c}b}\, \partial_{\mathrm{II}}^{d\dot{e}} \psi^b_{\dot{e}}(x_{\mathrm{I}}, x_{\mathrm{II}}) \xi^p_r(z_{\mathrm{I}}, z_{\mathrm{II}}) = (\Phi_p(x_{\mathrm{I}}, x_{\mathrm{II}}) - M^2_m) \chi^d_{\dot{c}}(x_{\mathrm{I}}, x_{\mathrm{II}}) \xi^p_r(z_{\mathrm{I}}, z_{\mathrm{II}}) \quad \text{(A1)}$$

$$\Box_{\mathrm{I}} \Box_{\mathrm{II}}\, \Phi_p(x_{\mathrm{I}}, x_{\mathrm{II}}) = \tfrac{1}{2} \mathrm{Re}(\psi^a_{\dot{b}}(x_{\mathrm{II}}, x_{\mathrm{I}}) \chi^{\dot{b}}_a(x_{\mathrm{II}}, x_{\mathrm{I}})) \quad \text{(A2)}$$

$$x^\mu = x^\mu_{\mathrm{II}} - x^\mu_{\mathrm{I}}, \qquad X^\mu = (1 - a) x^\mu_{\mathrm{I}} - a x^\mu_{\mathrm{II}} \quad \text{(A3a)}$$

$$\partial_{\mathrm{I}}^{a\dot{b}} = (1 - a)(-\delta^{a\dot{b}} \partial_{X^0} - \bar{\bar{\sigma}}^{a\dot{b}} \bar{\bar{\partial}}_{\bar{X}}) + \delta^{a\dot{b}} \partial_0 + \bar{\bar{\sigma}}^{a\dot{b}} \bar{\bar{\partial}}$$

$$\partial_{\mathrm{II}\dot{e}f} = a(-\delta_{\dot{e}f} \partial_{X^0} + \bar{\bar{\sigma}}_{\dot{e}f} \bar{\bar{\partial}}_{\bar{X}}) - \delta_{\dot{e}f} \partial_0 + \bar{\bar{\sigma}}_{\dot{e}f} \bar{\bar{\partial}}$$

$$\partial_0 = \partial / \partial x^0, \qquad \bar{\bar{\partial}} = \partial / \partial \bar{\bar{x}} \quad \text{(A3b)}$$

Here, x_{I} and x_{II} are the quark coordinates, z_{I} and z_{II} are the internal quark coordinates, x is the relative coordinate of the quarks, and X is the laboratory coordinate of the meson with $\mu = 0, 1, 2, 3$. χ and ψ are meson wave functions each consisting of a four-vector in the form of (I 6.11) and (I 8.1b). ξ_r^p is the internal function for the meson characterizing its internal properties via its flavors p and r. Φ_P is the interquark potential, which depends only upon $\mathbf{x}$ here. M_m is the average mass of the both quarks. Equation (A1) has been converted into an action integral (3.1) of ref. 14:

$$S_M = \int d^4x_{\mathrm{I}}\, d^4x_{\mathrm{II}}\, L_M \tag{A4a}$$

$$\begin{aligned} L_M = {} & \tfrac{1}{4}\{(\partial_{\mathrm{I}}^{\dot b a}\chi_a^{\dot e})(\partial_{\mathrm{II}\dot e f}\chi_{\dot b}^{f}) + (\partial_{\mathrm{II}}^{\dot b a}\psi_a^{\dot e})(\partial_{\mathrm{I}\dot e f}\psi_{\dot b}^{f}) + \text{h.c.}\} \\ & + \tfrac{1}{2}(\Phi_p(\bar{\bar{x}}) - M_m^2)(\psi_d^{\dot c}\chi_{\dot c}^{d} + \text{h.c.}) \end{aligned} \tag{A4b}$$

where $\chi_a^{\dot e} = (\chi_{\dot a}^{e})^*$. The ξ's have been removed by virtue of the orthonormal condition (2.6) and (2.7) of ref. 16:

$$\xi_p^r(z_{\mathrm{I}}, z_{\mathrm{II}})\xi_r^\rho(z_{\mathrm{I}}, z_{\mathrm{II}}) = 1, \qquad \xi_p^r = (\xi_r^\rho)^* \tag{A5a}$$

$$z_{\mathrm{I}}^p z_{\mathrm{I}r} = z_{\mathrm{II}}^p z_{\mathrm{II}r} = \delta_r^R, \qquad z_{\mathrm{I}}^p z_{\mathrm{II}r} = 0 \tag{A5b}$$

U(1) gauge transformation of (A4) has been carried out and the associated invariance shown in Section 4 of ref. 14. In the presence of internal coordinates, this transformation is generalized to

$$\partial_{\mathrm{I}}^{a\dot b}\chi_{\dot b}^{f} = ((1-a)\partial_X^{a\dot b} - \partial^{a\dot b})\chi_{\dot b}^{f} \rightarrow ((1-a)(\partial_X^{a\dot b} + iq_{\mathrm{I}}A^{a\dot b}(X)) - \partial^{a\dot b})\chi_{\dot b}^{f} \tag{A6a}$$

$$\partial_{\mathrm{II}a\dot f}\chi_{b}^{\dot f} = (a\partial_{Xa\dot f} + \partial_{a\dot f})\chi_{b}^{\dot f} \rightarrow (a(\partial_{Xa\dot f} - iq_{\mathrm{II}}A_{a\dot f}(X)) + \partial_{a\dot f})\chi_{b}^{\dot f} \tag{A6b}$$

$$q_{\mathrm{I}} = \sum_{\nu} q_\nu(z_{\mathrm{I}}^{\nu}\partial/\partial z_{\mathrm{I}}^{\nu} - z_{\mathrm{I}\nu}\,\partial/\partial z_{\mathrm{I}\nu}), \qquad \mathrm{I} \rightarrow \mathrm{II} \tag{A6c}$$

$$q_{\mathrm{I}} = q_4 = 2e/3, \qquad q_{\mathrm{I}} = q_3 = q_5 = -e/3 \tag{A6d}$$

$$\begin{aligned} & \chi_{\dot b}^{f} \rightarrow \chi_{\dot b}^{f}\exp(iq_{\mathrm{I}}\phi_q(X)), \qquad \chi_{b}^{\dot f} \rightarrow \chi_{b}^{\dot f}\exp(-iq_{\mathrm{II}}\phi_q(X)) \\ & A^{a\dot b}(X) \rightarrow A^{a\dot b}(X) - \partial_X^{a\dot b}\phi_q(X) \end{aligned} \tag{A6e}$$

where (A3b) has been consulted. With (A6c), which is (2.8a) of ref. 16, the meson wave functions are attached by the associated internal functions

$$\chi_{\dot b}^{f} \rightarrow \chi_{\dot b}^{f}\xi_r^p(z_{\mathrm{I}}, z_{\mathrm{II}}), \qquad \chi_{b}^{\dot f} \rightarrow \chi_{b}^{\dot f}\xi_p^r(z_{\mathrm{I}}, z_{\mathrm{II}}) \tag{A7}$$

as in (A1). Equation (4.7) of ref. 16,

$$\xi_r^p(z_{\mathrm{I}}, z_{\mathrm{II}}) = \sqrt{1/2}(z_{\mathrm{I}}^p z_{\mathrm{II}r} + z_{\mathrm{II}}^p z_{\mathrm{I}r}) \tag{A8}$$

comes from (I 9.1a) and the symmetric quark hypothesis (I 9.2) and refers to mesons at rest. In V $\rightarrow$ Pγ here, V is at rest and P is in motion, but both contain the same quarks. Therefore, (I 9.2) cannot be applied unambiguously. This ambiguity is removed if (A8) is replaced by

$$\xi_r^p(z_{\mathrm{I}}, z_{\mathrm{II}}) = z_{\mathrm{I}}^p z_{\mathrm{II}r} \text{ (or } z_{\mathrm{II}}^p z_{\mathrm{I}r}) \tag{A9}$$

which is an eigenfunction of (A6c). For consistency, this replacement should also be carried out for (4.7) of ref. 16. It then leads to $q_{\mathrm{I}}+q_{\mathrm{II}}=e$ below (4.7) of ref. 16 and the predicted vector meson magnetic moments there should be halved.

APPENDIX B. GENERAL MESON WAVE FUNCTIONS AND NONRELATIVISTIC APPROXIMATION

B.1. Meson Wave Equations in Vector Form

For an ansatz of type (3.1), (A1) has been put in vector form by means of (A3) and becomes (I 6.4) and the accompanying sister equation. These, including correction of some misprints there, are reproduced below:

$$\begin{aligned}
&[a(1-a)(E^2+\bar{K}^2)+\partial_0^2+\nabla^2+i(1-2a)(E\partial_0-\bar{K}\bar{\partial})]\chi \\
&\quad+[2\partial_0\bar{\partial}+i(1-2a)(E\bar{\partial}-\bar{K}\partial_0)-2a(1-a)E\bar{K}+\bar{K}\times\bar{\partial}]\bar{\sigma}\chi \\
&\quad+[a(1-a)(E^2-\bar{K}^2)+\partial_0^2-\nabla^2 \\
&\quad+i(1-2a)(E\partial_0+\bar{K}\bar{\partial})]\bar{\sigma}\bar{\chi}+E\bar{\sigma}(\bar{\partial}\times\bar{\chi}) \\
&\quad+[2\bar{\sigma}\bar{\partial}+2\partial_0+i(1-2a)(E-\bar{\sigma}\bar{K})]\bar{\partial}\bar{\chi} \\
&\quad-[i(1-2a)(\partial_0+\bar{\sigma}\bar{\partial})+2a(1-a)(E-\bar{\sigma}\bar{K})]\bar{K}\bar{\chi} \\
&\quad+(\bar{\partial}+\partial_0\bar{\sigma})(\bar{K}\times\bar{\chi})=(\Phi_p-M_m^2)(\psi-\bar{\sigma}\bar{\psi})
\end{aligned} \tag{B1a}$$

(B1a) with $\chi \leftrightarrow \psi$, cross products change sign

$$E = E_{J\bar{K}}, \qquad \chi, \bar{\chi}, \psi, \bar{\psi} = \chi_{J\bar{K}}, \bar{\chi}_{J\bar{K}}, \psi_{J\bar{K}}, \bar{\psi}_{J\bar{K}} \tag{B1b}$$

$$\bar{K} = \bar{K}_J, \qquad \Phi_P = \Phi_{PJ}, \qquad \nabla^2 = \bar{\partial}\bar{\partial} \tag{B1c}$$

$$a = 1/2 + \omega_{0\bar{K}}/E_{0\bar{K}} \tag{B1d}$$

In the rest frame, $\mathbf{K}_J = 0$ in (3.1), and in the absence of orbital excitation, (B1) and (A2) reduce to simple radial equations (7.3), (7.4), (8.3), and (8.4) of I. For the relative energy $\omega_{J\mathbf{K}} = 0$ mentioned below (4.1) and (B9), (B1d) yields $a = 1/2$. Making use of Φ_{PJ} (III 2.1b) with Φ'_{PJn} and $e_m = 0$ according to Sections 2 and 2.1 of III, (B1a) becomes

$$\begin{aligned}
&\{-\tfrac{1}{4}(E^2+\bar{K}^2)\chi-\tfrac{1}{2}E\bar{K}\bar{\chi}\}+[\nabla^2\chi-\bar{\partial}(\bar{K}\times\bar{\chi})] \\
&\quad=(d_m/r-\Phi_0-M_m^2)\psi
\end{aligned} \tag{B2a}$$

$$\{-\tfrac{1}{4}(E^2 - \bar{K}^2)\bar{\chi} + \tfrac{1}{2}\bar{K}(\bar{K}\bar{\chi}) + \tfrac{1}{2}E\bar{K}\chi\}$$
$$+ [2\bar{\partial}(\bar{\partial}\bar{\chi}) - \nabla^2\chi + E(\bar{\partial} \times \bar{\chi}) + \bar{K} \times \bar{\partial}\chi]$$
$$= -(d_m/r - \Phi_0 - M_m^2)\bar{\psi} \tag{B2b}$$

B.2. Extremely Relativistic Mesons

In the opposite limit, $\mathbf{K}_J=(0,\ 0,\ K \rightarrow \infty)$, the following dimensional considerations are given. The K^2-order terms in the braces of (B2) yield (I 6.12a).

$$\omega_{J\bar{K}} = 0, \qquad \chi_3 = -\chi, \qquad \mathrm{E}^2_{J\bar{K}} = K^2 \tag{B3}$$

Since the right sides of (B2) can be dropped, the brackets in (B2b) then show that

$$\left|\bar{\partial}\right| \approx K \tag{B4a}$$

so that these brackets are also of order K^2. This may alter the second relation of (B3), but the third one is assumed to hold. Equation (B4a) implies that

$$\chi,\ \bar{\chi},\ \psi,\ \bar{\psi},\ \approx \text{(finite power polynomial in } \bar{x}) * \exp(-\,Kr) \tag{B4b}$$

Their amplitudes are determined from the same normalization condition (III 3.7) and are thus of the order (III 2.3) with $d_m \rightarrow 2K$.

Should solutions of this type exist, they imply a decrease of the size of mesons at rest, $r_0 \cong 1$ fm (III 2.6a), to the order of $1/K \ll 2/d_m$ at high momenta. This reduction is in the relative space **x** and parallels the corresponding Lorentz contraction in laboratory space **X**.

This phenomenon makes it possible for mesons to be used to probe the hadronic structure of nucleons, similar to the use of electrons to probe their electromagnetic structure. For this purpose, meson energies $K \gg d_m/2 = 0.432$ GeV as well as the inverse of the nucleon size are needed.

In the intermediary region, $0 < K < \infty$, (B2) has not been solved.

B.3. Nonrelativistic Pseudoscalar Mesons

For pseudoscalar mesons moving nonrelativistically, such as D and B in Table I, the slow-meson approximation

$$\varepsilon_0 = K_0/E_{00} \ll 1 \tag{B5}$$

provides a small parameter and (B1) can be treated iteratively. Let

$$\chi_{0\bar{K}} = \sum_i \chi_{0i}, \qquad \bar{\chi}_{0\bar{K}} = \sum_i \bar{\chi}_{0i}, \qquad \chi \rightarrow \psi, \qquad E_{0\bar{K}} = \sum_i E_{0i} \tag{B6}$$

where the subscript i denotes the ith order in ε_0. For free mesons, Φ_P is independent of the meson wave functions according to Section 2.1 of III. To zeroth order, I Section 6 shows that

$$\chi_{00} = -\psi_{00}, \qquad \bar{\bar{\chi}}_{00} = \bar{\bar{\psi}}_{00} = 0 \tag{B7}$$

for pseudoscalar mesons at rest. To first order in ε_0, (B1) reduces to

$$\chi_{01} = 0, \qquad \psi_{01} = 0, \qquad E_{01} = 0 \tag{B8a}$$

$$\begin{aligned}(\tfrac{1}{4}E_{00}^2 - \nabla^2)\bar{\bar{\chi}}_{01} + 2\bar{\bar{\partial}}(\bar{\bar{\partial}}\bar{\bar{\chi}}_{01}) + E_{00}\bar{\bar{\partial}} \times \bar{\bar{\chi}}_{01} + (d_m/r - \Phi_0 - M_m^2)\bar{\bar{\psi}}_{01}\\ = \tfrac{1}{2}E_{00}\bar{\bar{K}}\chi_{00} - \bar{\bar{K}} \times \bar{\bar{\partial}}\chi_{00}\end{aligned} \tag{B8b}$$

$$\begin{aligned}(\tfrac{1}{4}E_{00}^2 - \nabla^2)\bar{\bar{\psi}}_{01} + 2\bar{\bar{\partial}}(\bar{\bar{\partial}}\bar{\bar{\psi}}_{01}) - E_{00}\bar{\bar{\partial}} \times \bar{\bar{\psi}}_{01} + (d_m/r - \Phi_0 - M_m^2)\bar{\bar{\chi}}_{01}\\ = \tfrac{1}{2}E_{00}\bar{\bar{K}}\psi_{00} + \bar{\bar{K}} \times \bar{\bar{\partial}}\psi_{00}\end{aligned} \tag{B8c}$$

It can be seen here that $\omega_{0\mathbf{K}}$ can at most be of order ε_0, which is consistent with (B3). To second order in ε_0, the singlet part of (B1) reads

$$\begin{aligned}&(\tfrac{1}{4}E_{00}^2 + \nabla^2)\chi_{02} - (d_m/r - \Phi_0 - M_m^2)\psi_{02}\\ &\quad = -i2\frac{\omega_{0\bar{\bar{K}}}}{E_{00}}\overline{\bar{\bar{K}}\bar{\bar{\partial}}}\chi_{00} - \bar{\bar{\partial}}(\bar{\bar{K}} \times \bar{\bar{\chi}}_{01}) - (\tfrac{1}{2}E_{00}E_{02} + \tfrac{1}{4}\bar{\bar{K}}^2)\chi_{00}\\ &\qquad - \tfrac{1}{2}E_{00}\overline{\bar{\bar{K}}\bar{\bar{\chi}}_{01}}\end{aligned} \tag{B9a}$$

$$\begin{aligned}&(\tfrac{1}{4}E_{00}^2 + \nabla^2)\psi_{02} - (d_m/r - \Phi_0 - M_m^2)\chi_{02}\\ &\quad = -i2\frac{\omega_{0\bar{\bar{K}}}}{E_{00}}\overline{\bar{\bar{K}}\bar{\bar{\partial}}}\psi_{00} + \bar{\bar{\partial}}(\bar{\bar{K}} \times \bar{\bar{\psi}}_{01}) - (\tfrac{1}{2}E_{00}E_{02} + \tfrac{1}{4}\bar{\bar{K}}^2)\psi_{00}\\ &\qquad - \tfrac{1}{2}E_{00}\overline{\bar{\bar{K}}\bar{\bar{\psi}}_{01}}\end{aligned} \tag{B9b}$$

Again, $\omega_{0\mathbf{K}}$ can be of order ε_0 or higher, in agreement with the first-order result above. However, $\omega_{0\mathbf{K}} \approx \varepsilon_0$ will lead to complex second-order wave functions and introduction of a new unknown. This is avoided if $\omega_{0\mathbf{K}} = 0$, as is required by the discussion preceding (4.2). In this way, the triplet part in (B1) to order ε_0^2 will have no source and hence drop out.

B.4. Classical Energy-Momentum Relation

The classical relation

$$E_{J\bar{\bar{K}}}^2 = E_{J0}^2 + \bar{\bar{K}}_J^2 \tag{B10}$$

holds in the $K_J \to \infty$ limit by (B3) and for $K_J = 0$ by definition. To first order in K_J or ε_0, (B10) also holds according to (B8a) together with $E_{11} =$

0, which can be verified in an analogous manner. To order ε_0^2, (B10) has not been established generally.

B.5. Perturbed Wave Functions from Dimensional Analyses for Heavy Mesons

Even (B8) and (B9) are not simply solved because the spherical symmetry present in the $\varepsilon_0 = 0$ limit is broken by the motion $\mathbf{K}_0$ so that separation of variables in the relative space $\mathbf{x}$ cannot be carried out. Therefore, (B8) and (B9) will be treated by a dimensional analysis, which is further much simplified for heavy mesons like D and B in Table I:

$$E_{00} \gg \; 2\left|\bar{\partial}\psi_{00}/\partial r\right|/\psi_{00} = d_m = 0.864 \text{ GeV} \tag{B11}$$

The opposite of (B11) holds for the light mesons π and K in Table I, for which, however, $\varepsilon_0 > 1$, so that Appendix B.3 above no longer holds. Under these conditions, (B8b), (B8c), and (B9) become

$$\bar{\chi}_{01}(\bar{x}) = -\bar{\psi}_{01}(\bar{x}) \tag{B12a}$$

$$(\tfrac{1}{4}E_{00}^2 - \nabla^2)\bar{\psi}_{01} + 2\bar{\partial}(\bar{\partial}\bar{\psi}_{01}) - (d_m/r - \Phi_0 - M_m^2)\bar{\psi}_{01} = \tfrac{1}{2}E_{00}\bar{K}\psi_{00} \tag{B12b}$$

$$-E_{00}\bar{\partial}\times\bar{\psi}_{01} = \bar{K}\times\bar{\partial}\psi_{00} \to 0 \tag{B12c}$$

$$\chi_{02}(\bar{x}) = -\psi_{02}(\bar{x}) \tag{B13a}$$

$$(\tfrac{1}{4}E_{00}^2 + \nabla^2)\psi_{02} + (d_m/r - \Phi_0 - M_m^2)\psi_{02} = -\left(\tfrac{1}{2}E_{00}E_{02} + \tfrac{1}{4}\bar{K}^2\right)\psi_{00} - \tfrac{1}{2}E_{00}\bar{K}\bar{\psi}_{01} \tag{B13b}$$

$$\bar{\partial}(\bar{K}\times\bar{\psi}_{01}) \to 0 \tag{B13c}$$

The procedure of the dimensional analysis approximation is to replace the operator of $\bar{\psi}_{01}$ in (B12b) by some scale constant, which is then fixed by (B12c). The result is

$$-\bar{\chi}_{01}(\bar{x}) = \bar{\psi}_{01}(\bar{x}) = \bar{K}\psi_{00}/E_{00} \tag{B14}$$

where the free meson wave function (III 2.2a) has been used for the zeroth-order wave function ψ_{00}. The choice

$$\bar{K} = (0, 0, K) \tag{B15}$$

entails no loss of generality. Self-consistency of (B12) is obtained for

$$-\bar{\chi}_{01} = \bar{\psi}_{01} = (0, 0, \psi_{01z}(x, y)) \tag{B16}$$

Here, x and y are relative space coordinates. Note that $\bar{\psi}_{01}$ in (B16) depends only upon x and y, while $\bar{\psi}_{01}$ in (B14) also depends upon z via $\psi_{00}(r)$. This inconsistenciy is regarded as part of the approximation.

Following the procedure leading to (B14), the operator for ψ_{02} in (B13b) is replaced by a scale constant. However, (B13c) is an identity by (B15) and (B16) and hence cannot fix this constant. Now, the left operator in (B13b) is the same as that for the zeroth-order ψ_{00} (I 6.10), so that the homogeneous part of ψ_{02} is proportional to ψ_{00}. The right side of (B13b) is also proportional to ψ_{00} by (B14). Therefore, the scale constant is estimated to be $4/d_m^2$, the square of the scale of ψ_{00} in (III 2.2a). This result, together with (B14) and (B10), the third of (B6), and (B8a), converts (B13a) and (B13b) to

$$-\chi_{02} = \psi_{02} = -4K^2\,\psi_{00}/d_m^2 \tag{B17}$$

It is pointed out that the dimensional analysis results (B14), (B16), and (B17) can only give order-of-magnitude estimates of the perturbed wave functions. The detailed depedence of these functions on $\mathbf{x}$ is lost in the approximation, as noted below (B16).

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